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Void analysis of target residues at SPS energy – evidence of correlation with fractal behaviour

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Abstract This paper presents an analysis of the target residues in ^{32}S – AgBr and ^{16}O – AgBr interactions at 200 AGeV and 60AGeV respectively in terms of fractal moment by Takagi method and void probability scaling The study reveals a interesting feature of the production process In ^{16}O – AgBr interactions multifractal behaviour is present in both hemispheres and void probability does not show a scaling behaviour, but at high energy the situation changes In ^{32}S – AgBr interactions for both hemisphere monofractal behaviour is indicated by that data and void probability also shows good scaling behaviour This suggests that a possible correlation of void probability with fractal behaviour of target residues

Keywords Multifractality, monofractality, fractal dimensions, void probability, forward hemisphere, backward hemisphere, fluctuations

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1. Introduction

For scrutinizing multifractality in high-energy multiparticle production process, Hwa first suggested the fractorial moment method [1-3] These fractal moments, statistically known as frequency moments, were extensively used to investigate whether multiparticle possesses fractal properties [4-7]. The scaling property of fractal moment can also be interpreted in terms of fractal properties of particle density fluctuations.

Apart from studying fractal properties of particle density fluctuations, the statistical counting variable, the scaled factorial moment is the main tool for detecting large dynamical fluctuations and investigating the pattern of fluctuation.

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But the scaled factorial moment method has a common disadvantage – the experimental data sets do not show the linearity in a log-log plot of moment against bin size as expected from the mathematical formulations. This may be due to the fact that the assumed mathematical limit is not valid for the real experimental data where number of particles in each event is always finite. The difficulty has been overcome in Takagi Moment method.

Now in high-energy physics the multiparticle production process is investigated extensively through the study of both fluctuation and correlation. The correlation can give direct information about the late stage of the reaction when nuclear matter is highly excited and diffused. Generally, two and three particle correlation are studied. But the multiparticle production in high energy collisions is one of the rare fields of physics where higher order correlation are directly accessible in their full dimensional characteristics under well controlled experimental conditions. Actual higher order correlations have been studied using UA1 data [8] and Na22 data [9] in terms of cumulant correlation function. However, the data on higher order correlation are scanty. In this context, it will be interesting to study higher order correlation through the void probability because that probability have some important features to make it very useful to study correlations of higher orders. Void probability is defined as the probability of occurrence of events with no particle in a specified region of phase space. The distribution of voids and multiplicity moments of all orders are known to be intimately linked, and the study of one can reveal information about the other.

In an analysis on short range two particle correlation [10] it was pointed out that higher order cumulant correlation function could be expressed in terms of the two particle cumulant correlation function. This investigation has suggested that the two particle cumulant correlation function can be written in terms of the inclusive single and two particle densities ρ_1 and ρ_2 as $C_2(X_1, X_2) = \rho_2(X_1, X_2) - \rho_1(X_1)\,\rho_1(X_2)$ [10]. The two particle reduced cumulants can be expressed as

$$c_2(X_1, X_2) = C_2(X_1, X_2)/\rho_1(X_1)\,\rho_1(X_2)\,.$$

Further it was conjectured that the reduced cumulants of order greater than or equal to three could be expressed in terms of linked pairs of two particle cumulants. In general c_N is thus proportional to the product of (N–1) two particle reduced cumulants summed over all permutations [10].

$$c_N(X_1, X_2, \ldots\ldots X_N) = A_N \sum_{perm} \prod^{N-1} c_2(X_i, X_j)\,. \tag{1}$$

where A_N is the linking coefficient of order N. Groth and Peebles [11-12] studying the two-dimensional galaxy catalogs, proved the above scheme to be successful in describing galaxy-galaxy correlations. The same relation has been suggested independently by Mandelbrot [13] on the theoretical grounds. In multiparticle phenomenology the above linking scheme (linked pair ansatz) for the N particle cumulant correlation function is known as linked pair approximation.

If such a relation of linked pair ansatz holds good, the void probability will follow certain scaling relations. Hegyi [14] while investigating the void probability in pseudo rapidity space, found a scaling behaviour, which implies the validity of the linked-pair approximation.

A good number of papers have been reported so far where a fractal study of pion spectra was performed. Also void probability study was also made with same data. However the detail investigation of fractality and void probability in case of target residues has not been attempted so far.

In view of this in this paper we have investigated the fractal nature of target residues [Takagi's method] in the interactions initiated by ^{32}S and ^{16}O at an energy value of 200 and 60 AGeV for full space, forward hemisphere and backward hemisphere separately in emission angle space. Also a study of higher order correlation through the void probability in the same interactions and also in the same space have been performed. The study reveals a possible correlation among the fractal nature and void probability scaling.

2. Experimental details

The data were obtained by exposing 200 AGeV sulphur beam and 60AGeV oxygen beam on Ilford G5 emulsion stacks at CERN SPS [15]. A Leitz Metalloplan microscope with a 10X objective and ocular lens provided with a semi-automatic scanning stage is used to scan the plates. Each plate is scanned by two independent observers to increase the scanning efficiency. For measurement 100X oil-immersion objective is used. The measuring system fitted with it has 1 μm resolution along the X and Y axes and 0.5 μm resolution along the Z axis.

Details of events selection criteria and classification of tracks can be found from our earlier communication [16].

For our present analysis we have considered 207 events in "S–AgBr interactions at 200 AgeV [17] and 250 events of ^{60}O–AgBr interactions at 60AGeV [18]. We have considered black tracks only because they represents the target residues.

The emission angle (θ) with respect to the beam direction are measured for each black track by taking the coordinates of the interaction point (X_0, Y_0, Z_0), coordinates (X_1, Y_1, Z_1) at the end of the linear portion of each secondary track and coordinates (X_0, Y_i, Z_i) of a point on the incident beam.

Nuclear emulsion covers 4π geometry and provides very good accuracy in the measurements of angles of produced particles and fragments due to high spatial resolution and thus, is suitable as a detector for the study of fluctuations in the fine resolution of the phase space considered.

3. Method of analysis

a. Multifractal moments using the Takagi method :

The selected phase space interval of length x has been divided into M bins of equal size, the width of each bin being $\delta x = x/M$. Then the multiplicity distribution for a single bin is denoted as $P_n(\delta x)$ for n = 0, 1, 2, *etc.* where we assume that the inclusive particle distribution dn/dx is constant and $P_n(\delta x)$ is independent of the location of the bin. n target residues, contained in a single event, is distributed in the interval $x_{min} < x < x_{max}$. The multiplicity n changes from event to event according to the distribution $P_n(x)$, where $x = x_{max} - x_{min}$. If the number of independent event is Ω, then the particle produced in those events are distributed in ΩM bins of size δx. Let N be the total number of target associated slow particles produced in these Ω event and n_{aj} the multiplicity of black particles in the j^{th} bin of the a^{th} event.

The theory of multifractals [19,20] has been motivated to consider the normalized density P_{aj} defined by $P_{aj} = n_{aj}/N$.

This is of course also true when $N \to \infty$. Then one has to consider the Takagi moment of order q as

$$T_q(\delta x) = \ln \sum_{a=1}^{\Omega} \sum_{j=1}^{M} P_{aj}^q$$

which behaves like a linear function of the logarithm of the "resolution" $R(\delta x)$

$$T_q(\delta x) = A_q + B_q \ln R(\delta x)$$

where A_q and B_q are constants independent of δx. If such a behavior is observed for a considerable range of $R(\delta x)$, a generalised dimension may be determined as

$$D_q = \frac{B_q}{q-1}. \tag{2}$$

Now evaluating the double sum of P_{aj}^q for sufficiently large Ω, a linear relation is expected [21]

$$\ln\langle n^q \rangle = A_q + (B_q + 1)\ln R(\delta x)$$

While analyzing real data [22] it was observed [23] that plot of $\ln\langle n^q \rangle$ against δx saturates for large x region. This deviation may be due to the nonflat behaviour of dn/dx in the large x region. Takagi suggested that $\langle n \rangle$ would be a better choice of the "resolution" $R(\delta x)$ because $dn/\langle n \rangle$ is flat by definition. Choosing $R(\delta x) = \langle n \rangle$ one has

$$\ln\langle n^q \rangle = A_q + (B_q + 1)\ln\langle n \rangle \tag{3}$$

a simple linear relation between $\ln\langle n^q \rangle$ and $\ln\langle n \rangle$. This <..> symbol indicates averaging over events. The generalised dimension D_q can be obtained from the slope values using eq. (1).

The case with $q = 1$ can be obtained by taking an appropriate limit [20]. The value of information dimension D_1 can also be determined from a new and simple relation suggested by Takagi

$$\frac{\langle n \ln n \rangle}{\langle n \rangle} = C_1 + D_1 \ln\langle n \rangle \tag{4}$$

where C_1 is a constant.

b. Scaled gap probability :

For gap analysis in emission angle space the method enunciated by Hegyi [14] is followed. If $Q(\lambda)$ is the probability generating function for $P_n(\Delta\theta)$,

$$Q(\lambda) - \sum_{n=0}^{\infty} (1-\lambda)^n \; P_n(\Delta\theta) \tag{5}$$

where $P_n(\Delta\theta)$ is the probability of detecting exactly n particles $\Delta\theta$ in region.

Mueller [24] and White [25] proposed that the above equation can be expressed as the integral form of N particle correlation functions in a similar way to cluster expansion of statistical mechanics. In terms of the reduced factorial cumulant $\bar{K}_N$, $Q(\lambda)$ can be written as,

$$Q(\lambda) = \exp \; {}_{N=1} \; \frac{(-\lambda\bar{n})^N}{N!} \; \bar{K}_{\Lambda} \tag{6}$$

With $\bar{n}$ is the average number of particles in $\Delta\theta$ region, the probability of finding no particles in $\Delta\theta$ is related to the generating function through the relation

$$P_0(\Delta\theta) = Q(\lambda = 1)\,. \tag{7}$$

The emission angle gap probability $P_0(\Delta\theta)$ can be used as a generating function $Q(\lambda)$ for P_n [26, 27] as

$$P_n(\Delta\theta) = \frac{(-\bar{n})^n}{n!}\left(\frac{\partial}{\partial\bar{n}}\right)^n P_0(\Delta\theta)\,. \tag{8}$$

This equation expresses the relation between the n-particle and zero-particle probabilities in a region $\Delta\theta$. This equation has been obtained by allowing only $\bar{n}$ to vary in $P_0(\Delta\theta)$ and all other parameters of $P_0(\Delta\theta)$ are taken to be fixed with respect to the $\bar{n}$ variation. This important feature of the gap probability was first emphasized by White and discussed in details by Balian and Schaeffer [28]. Another property worth noticing that $P_0(\Delta\theta)$ is symmetrically dependent on the hierarchy of correlation function. Besides serving as a generating function gap probability is also related to the probability $P_n(\Delta\theta)$ with $n \neq 0$ through various kinds of moments. $P_0(\Delta\theta)$ can be written as an expansion in cumulants as

$$\ln P_0(\Delta\theta) = \sum_{N-1}^{\infty} \frac{(-\bar{n})^n}{N!} \bar{K}_N \tag{9}$$

Thus gap probability fundamental relationship to all of the probabilities $P_n(\Delta\theta)$ Furthermore $P_0(\Delta\theta)$ can provide a good discriminator between various theoretical models making predictions for higher order correlations.

If the cumulant correlation functions satisfy the linked pair ansatz, then the reduced factorial moment takes the form

$$\bar{K}_N = A_N\, K_2^{N-1} \tag{10}$$

We can define a parameter χ, with $P_0(\Delta\theta) = \exp(-\bar{n}\,\chi)$, called the scaled emission angle gap probability. Thus

$$\chi = -\ln P_0\,(\Delta\theta)/\bar{n}\,. \tag{11}$$

If the linking coefficient A_N becomes independent of the collision energy and bin size [29, 30], χ depends only on the single moment combination $\bar{n}\,\bar{K}_2$ so that we can write

$$\chi = \sum_{N=1}^{\infty} \frac{1}{N!} A_N \left(-\bar{n}\,\bar{K}_2\right)^{N-1} = \chi\left(-\bar{n}\,\bar{K}_2\right). \tag{12}$$

It is seen from equation (11) that for the poisson distribution $\bar{n} = -\ln P_0$. So $\chi = 1$. χ therefore can be interpreted as the gap probability which normalizes out the contribution from totally uncorrelated particle emission. The overall shape of the scaling function for $\chi < 1$ is affected only by the clustering properties of the secondaries involving correlation from all the orders. This feature makes the scaled gap probability well suited to investigate the production of target residues and their structure in the emission angle space.

Although expansion given in (11) is technically only valid for small values of $\bar{n}\,\bar{K}_2$, the implication for clustering do extend beyond this. For large values of $\bar{n}\,\bar{K}_2$, models with different hierarchical amplitudes, A_N gives different values of scaled gap probability χ. As $\bar{n}\,\bar{K}_2$ increases the value of χ gets smaller.

Results

In the present case, we have considered emission angle as the phase space variables and we have analyzed the data of target residues produced in $^{32}S - AgBr$ and $^{16}O - AgBr$ interactions at 200 AGeV and 60 AGeV respectively.

For the analysis of Takagi moment method, the cosine of emission angle interval is divided into overlapping bins, whose size is increased symmetrically in steps of 0.1 around the central value 0 (zero) for full space, 0.5 for forward hemisphere and – 0 5 for backward hemisphere. Now for each bin we have calculated $\langle n^q \rangle$ with q = 2, 3, 4, 5 and $\langle n \ln n \rangle / \langle n \rangle$ for all hemispheres of both the spaces in $^{32}S - AgBr$ and $^{16}O - AgBr$ interactions at 200 AGeV and 60 AGeV respectively. Figure 1(a) and (b) represent the nature of variation of

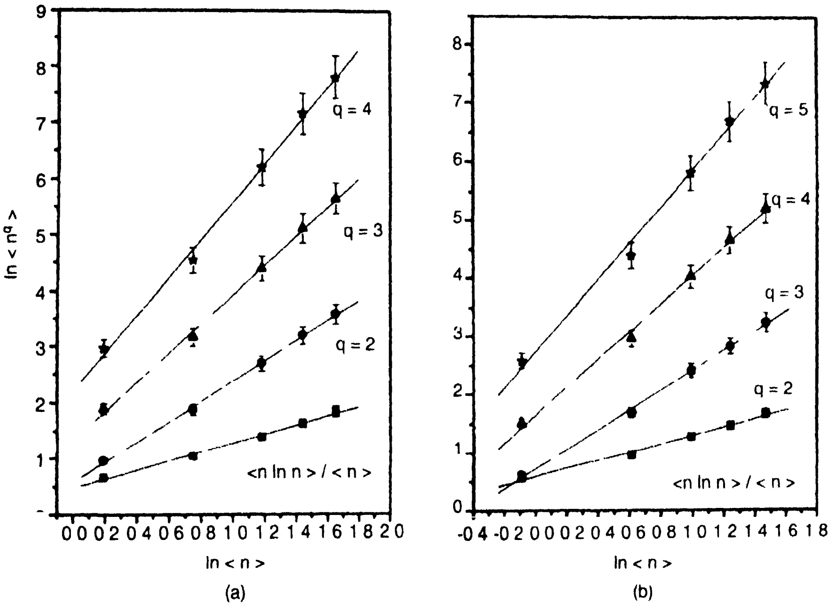

Figure 1. The dependence of $\ln\langle n^q \rangle$ on $\ln\langle n \rangle$ for order q = 2, 3, 4 and $\langle n \ln n \rangle / \langle n \rangle$ on $\ln\langle n \rangle$ for order q = 1 for (a) in forward hemisphere and (b) in backward hemisphere for $\cos\theta$ space in $^{32}S - AgBr$ interactions at 200 AGeV

$\ln\langle n^q \rangle$ with $\ln\langle n \rangle$ for q = 2, 3, 4, 5 and $\langle n \ln n \rangle / \langle n \rangle$ with $\ln\langle n \rangle$ for forward and backward hemispheres for $^{32}S - AgBr$ interactions respectively. Figure 2(a) and (b) represent the same for same hemispheres for $^{16}O - AgBr$ interactions.

Both $\langle n \ln n \rangle / \langle n \rangle$ and $\ln\langle n^q \rangle$ in $^{32}S - AgBr$ and $^{16}O - AgBr$ interactions exhibit a linear behavior as function of $\ln\langle n \rangle$. We have performed best linear fits to the data sets and using equation (1) and (2) we have calculated the values of generalized dimension D_q

Table 1 Parameters of Takagi moment analysis in $\cos\theta$ space for $^{32}S - AgBr$ interaction at 200 AGeV

Interaction	Space	Hemisphere	D_q		
			$q = 2$	$q = 3$	$q = 4$
		Full [31]	84	79	75
$^{32}S - AgBr$	$\cos\theta$	Forward	80 ± 03	80 ± 07	80 ± 12
		Backward	70 ± 04	70 ± 08	70 ± 14

Table 2. Parameters of Takagi moment analysis in $\cos\theta$ space for $^{16}S - AgBr$ interaction at 60 AgeV

Interaction	Space	Hemisphere	D_q		
			$q = 2$	$q = 3$	$q = 4$
		Full [31]	87	83	75
$^{16}O - AgBr$	$\cos\theta$	Forward	78 ± 01	76 ± 02	74 ± 11
		Backward	79 ± 02	78 ± 04	77 ± 06

given in Table 1 and Table 2 The information dimensions D_1 is obtained using equation (3) The values are listed in Table 1 and Table 2 for $^{32}S - AgBr$ and $^{16}O - AgBr$ interactions respectively. The values of D_q have been plotted against q in Figure 2(a) and (b) for $^{32}S - AgBr$ and $^{16}O - AgBr$ interactions respectively The results for full hemisphere in both interactions have given in our earlier work [31]

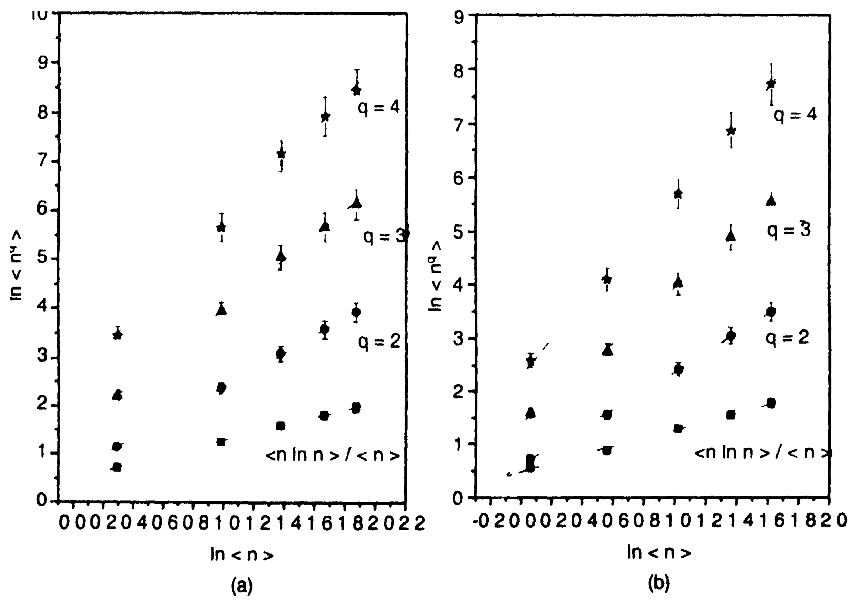

Figure 2. The dependence of $\ln\langle n^q\rangle$ on $\ln\langle n\rangle$ for order q = 2, 3, 4 and $\langle n \ln n\rangle/\langle n\rangle$ on $\ln\langle n\rangle$ for order q = 1 for (a) in forward hemisphere and (b) in backward hemisphere for $\cos\theta$ space in $^{16}S - AgBr$ interactions at 200 AGeV

Both $\langle n \ln n\rangle/\langle n\rangle$ and $\ln\langle n^q\rangle$ in all cases exhibit a linear behavior as function of $\ln\langle n\rangle$ We have performed best linear fits to the data sets and using equation (1) and (2) we have calculated the values of generalized dimension D_q given in Table 1 and Table 2 The information dimension D_1 is obtained using equation (3) The values are listed in

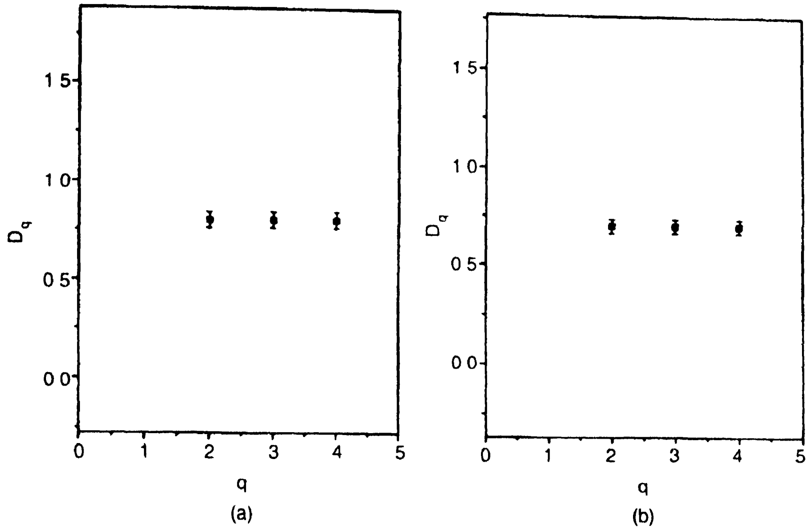

Figure 3. The q dependence of generalized dimension D_q for (a) in forward hemisphere and (b) in backward hemisphere for $\cos\theta$ space in $^{32}S - AgBr$ interactions at 200 AGeV

Table 1 and Table 2 for $^{32}S - AgBr$ and $^{16}O - AgBr$ interactions respectively The values of D_q have been plotted against q in Figure 4(a) and (b) and in 5(a), 5(b) and it is observed that the values of generalized dimension D_q decrease with the increase of order

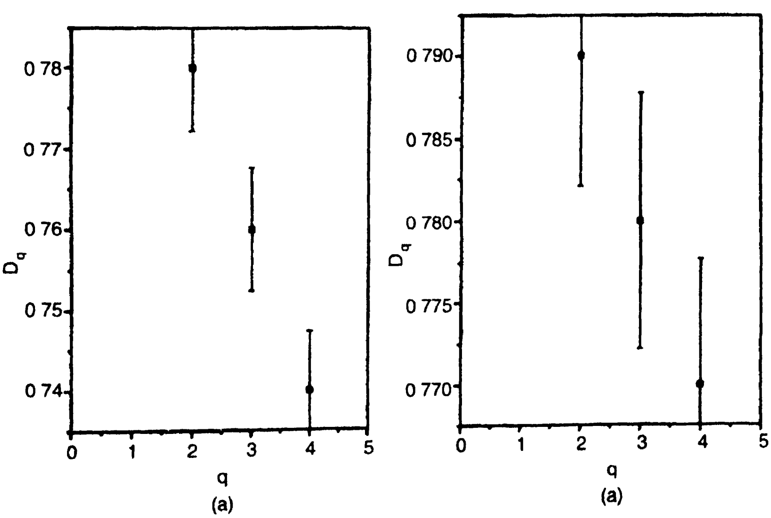

Figure 4. The q dependence of generalized dimension D_q for (a) in forward hemisphere and (b) in backward hemisphere for $\cos\theta$ space in $^{16}S - AgBr$ interactions at 60 AGeV

q in full hemisphere for $^{32}S - AgBr$ interactions but the same result has been observed for each hemisphere (full, forward, backward) in $^{16}O - AgBr$ interactions. These results reflect the presence of multifractal geometry.

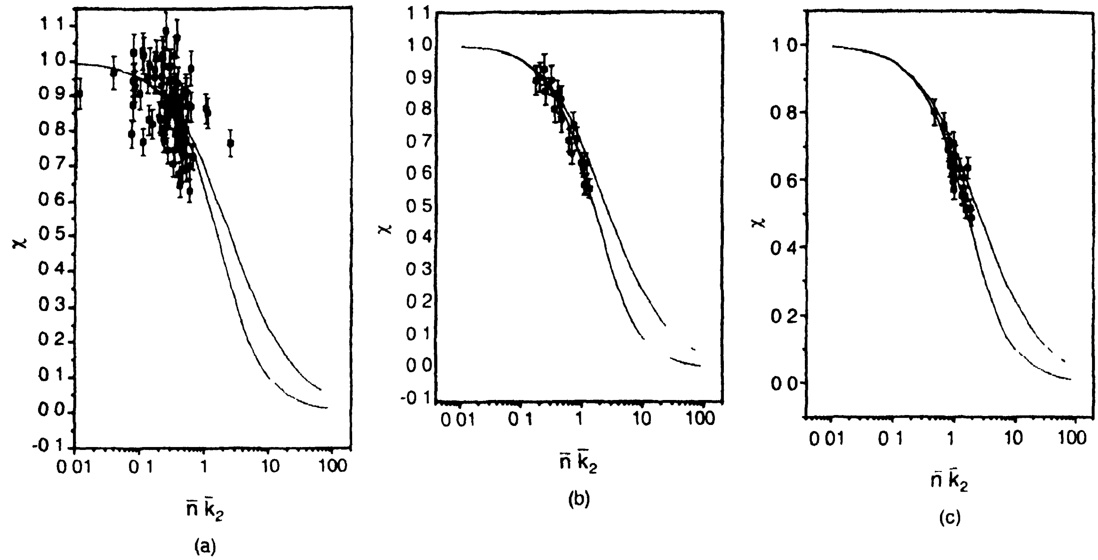

Figure 5. Void probability scaling for (a) in full hemisphere (b) in forward hemisphere (c) in backward hemisphere for $\cos\theta$ space in $^{32}S - AgBr$ interactions at 200 AGeV

But D_q gives a unique value for different orders of moments in forward and backward hemispheres in $^{32}S - AgBr$ interactions which reflects the presence of monofractal geometry.

Now we wanted to find the void probability P_0. For that in emission angle space we have taken the centre at 0 (bin centre) for full space, 0.5 for forward hemisphere and −0.5 for backward hemisphere. Then we have shifted the centre in steps of 0.001 units from 0 to 0.006 for each of the window sizes $\Delta\cos\theta$ = 0.001, 0.002, 0.003, 0.004, 0.005, 0.006 and separately found the void probability in each of the cases.

We have calculated the single moment combination $(\bar{n}\,\bar{K}_2)$ and scaled void probability (χ_{obs}) for each of the bin centers and window sizes and plotted the values of the above in figure 5(a), (b), (c) and 6(a), (b), (c) for the $^{32}S - AgBr$ interactions and $^{16}O - AgBr$ interactions respectively.

Moreover we assumed that the uncertainty in the measurement of emission angle does not introduce any systematic error in P_0 values. On the same figures we have also shown the values of λ^{obs} obtained from two hierarchical models of galaxy clasterization represented by the outer solid line (negative binomial model) and inner solid line (minimal model). To show the scaling of the data points bounded by the two curves we have plotted the two theoretical models in the same figures along with the experimental points for full, forward and backward hemispheres in $^{32}S - AgBr$ interaction and $^{16}O - AgBr$ interactions respectively.

From the figure it is evident that for forward and backward hemispheres in emission angle space in $^{32}S - AgBr$ interactions there are a nice scaling behaviour and we may say that void probability scaling holds good in the above two scases. We have also

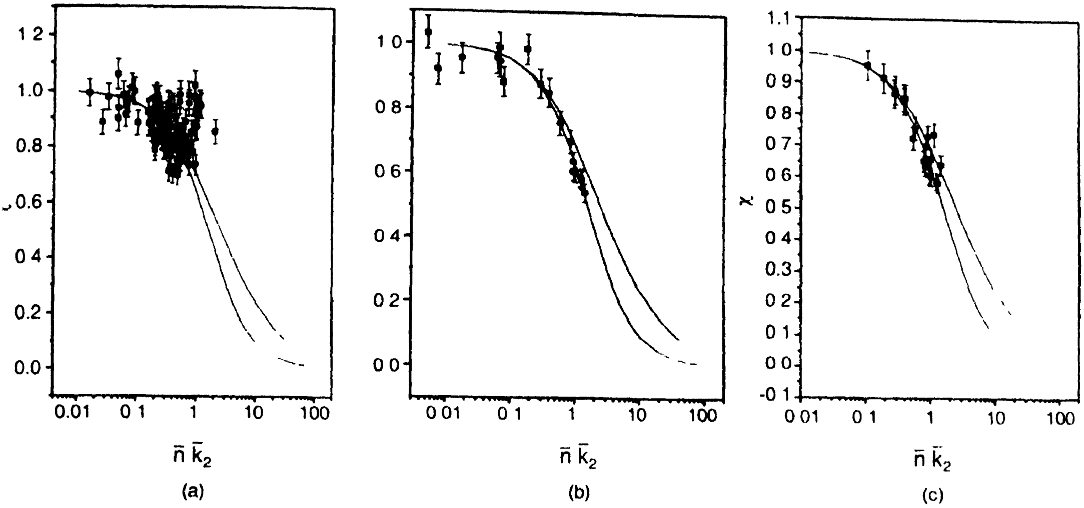

Figure 6. Void probability scaling for (a) in full hemisphere (b) in forward hemisphere (c) in backward hemisphere for $\cos\theta$ space in $^{16}O - AgBr$ interactions at 60 AGeV.

observed that void probability scaling is not seen in the full hemisphere in emission angle space of $^{32}S - AgBr$ interactions and in the all hemispheres for the same space in $^{16}O - AgBr$ interactions.

Therefore, from this study we have observed that in $^{16}O - AgBr$ interactions multifractal behaviour is present in both hemispheres and void probability does not show a scaling behaviour. But at high energy the situation changes. In $^{32}S - AgBr$ interactions in emission angle space for forward and for also backward hemisphere monofractal behaviour is indicated by that data and void probability also shows good scaling behaviour in that region.

Thus we may conclude that this investigation reveals a possible correlation between fractality and void probability scaling in case of target residues in relativistic interactions.

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